SOLUTION OF PROBLEMS 353 & 354, *IN NUMBER THREE.*

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353. "Required the average area of the circles described on the focal chords of a given ellipse as diameters."

SOLUTION BY PROF. E. B. SEITZ, KIRKSVILLE, MO.

Let $\theta =$ the angle which a focal chord makes with the major axis. Then the lengths of the two parts of the chord are
$\frac{a(1-e^2)}{1+e\cos\theta}$ and $\frac{a(1-e^2)}{1-e\cos\theta}$, and the length of the chord is $l=\frac{2a(1-e^2)}{1-e^2\cos^2\theta}$.
Hence the average area of the circle described on the chord is

$$\varDelta=\int_0^\pi \tfrac{1}{4}\pi l^2 d\theta \div \int_0^\pi d\theta = a^2\int_0^\pi \frac{(1-e^2)^2}{(1-e^2\cos^2\theta)^2}d\theta.$$

Let $\tan\theta = \sqrt{(1-e^2)}\tan\varphi$, then

$$\cos^2\theta = \frac{\cos^2\varphi}{1-e^2\sin^2\varphi},\quad \frac{1-e^2}{1-e^2\cos^2\theta} = 1-e^2\sin^2\varphi,$$

$$d\theta = \frac{\sqrt{(1-e^2)}\cos^2\theta d\varphi}{\cos^2\varphi} = \frac{\sqrt{(1-e^2)}d\varphi}{1-e^2\sin^2\varphi}.$$

$\varphi = 0$, when $\theta = 0$, and $\varphi = \pi$, when $\theta = \pi$; therefore

$$\varDelta = a^2(1-e^2)^{\frac{1}{2}}\int_0^\pi (1-e^2\sin^2\varphi)d\varphi = \pi ab(1-\tfrac{1}{2}e^2).$$

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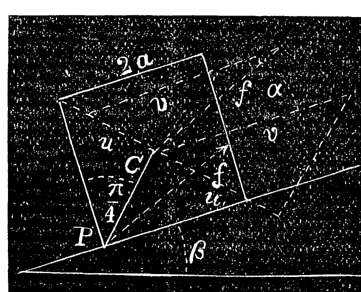
354. "A cube slides down an inclined plane with four of its edges horizontal. The middle point of its lowest edge comes in contact with a small fixed obstacle and is reduced to rest. Find the direction of the impulsive reaction of the obstacle, and show that it is independent of the velocity of the cube and of the inclination of the plane. Determine also the limiting velocity that the cube may be on the point of overturning."

SOLUTION BY PROF. H. T. EDDY, CINCINNATI, OHIO.



LET P be the opstacle and v the velocity of impact of the cube whose side is $2a$, whose mass is unity and whose radius of gyration squared $k^2=\frac{2}{3}a^2$. Also $PC = a\sqrt{2}$.

The velocity along PC is destroyed by the impact, after which C has a velocity u perpendic'r to PC and there is also rotary velocity w about C, such that $u = aw\sqrt{2}$.

Take the moments of momenta about P; $\therefore\ ua\sqrt{2}-va+k^2w=0$.

Substitute the values of u and k^2 in this equation, $\therefore\ aw=\frac{3}{8}v$, $\therefore\ v=v.\frac{3}{8}\sqrt{2}$.

The impulse f which must be combined with the initial momentum v in order that the cube may have the resultant momentum u is

$$f^2=v^2+u^2-2vu\cos\tfrac{1}{4}\pi=(1+\tfrac{18}{64}-\tfrac{6}{8})v^2;\ \therefore f=v.\tfrac{1}{8}\sqrt{34}.$$

To find the inclination a, we have $u^2=v^2+f^2-2vf\cos a$;

$$\therefore\ \cos a=5\div\sqrt{34},$$

which is *constant*.

That this is the correct result may be seen by showing that f applied at P in the direction a will cause the angular velocity w found above. For take moments about C; $k^2w=fa\sqrt{2}.\sin(\frac{1}{4}\pi-a)=fa(\cos a-\sin a)$;

$$\therefore\ \tfrac{2}{3}aw=\frac{\sqrt{34}}{8}v\left(\frac{5-3}{\sqrt{34}}\right)=\frac{v}{4};\ \therefore\ aw=\tfrac{3}{8}v,$$

which is the result previously obtained.

The initial velocity of the cube will be just sufficient to overturn it when the energy of translation and rotation are sufficient to raise the center C to a point directly above P; i. e., when

$$u^2+k^2w=2ga\sqrt{2}.[1-\cos(\tfrac{1}{4}\pi-\beta)];$$
$$\therefore\quad v^2=\tfrac{16}{3}\sqrt{2}.ag[1-\cos(\tfrac{1}{4}\pi-\beta)].$$

SOLUTIONS OF PROBLEMS IN NUMBER FOUR.

SOLUTIONS of problems in No. 4 have been received as follows:

From R. J. Adcock, 358; Prof. W. P. Casey, 355, 356, 358; George Eastwood, 356, 358; Prof. Asaph Hall, 358; Prof. E. B. Seitz, 355, 358; R. S. Woodward, 358. We also received a solution of 353, of No. 3, from the proposer, Mr. Hoover, and a solution of 352 from Mr. Heaton.

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355. "The length of a garden, in the form of a parallelogram, is one rod greater than the breadth. Within the garden is a fountain; and a gravel walk extends diagonally across the garden, from corner to corner, and the distance from the fountain to one end of said walk is three rods, and to the other end four rods; and from this end of the walk along one end of the garden, to the next corner, and from thence to the fountain, is eight rods. Required the area of the garden."